Int J Theor Phys (2010) 49: 759–765
DOI 10.1007/s10773-010-0254-0

Interacting New Agegraphic Phantom Model of Dark Energy in Non-flat Universe

M.R. Setare

Received: 22 October 2009 / Accepted: 15 January 2010 / Published online: 27 January 2010

Abstract In this paper we consider the new agegraphic model of interacting dark energy in non-flat universe. We show that the interacting agegraphic dark energy can be described by a phantom scalar field. Then we show this phantomic description of the agegraphic dark energy and reconstruct the potential of the phantom scalar field.

Keywords Phantom · New agegraphic · Interacting dark energy · Non-flat universe

1 Introduction

According to the cosmological observations our universe is undergoing an accelerating expansion, and the transition to the accelerated phase has been realized in the modern (recent) cosmological era [1–4]. In order to explain this remarkable behavior, and despite the intuition that this can be achieved only through a fundamental theory of nature, we can still propose some paradigms for its description. Thus, we can either consider theories of modified gravity [5–7], or introduce the concept of dark energy which provides the acceleration mechanism. The dynamical nature of dark energy, at least in an effective level, can originate from various fields, such as a canonical scalar field (quintessence) [8–12], a phantom field, that is a scalar field with a negative sign of the kinetic term [13–17], or the combination of quintessence and phantom in a unified model named quintom [18–28]. The advantage of this combined model is that although in quintessence the dark energy equation-of-state parameter remains always greater than -1 and in phantom cosmology always smaller than -1, in quintom scenario it can cross -1.

In addition, many theoretical studies are devoted to understand and shed light on dark energy, within the string theory framework. The Kachru-Kallosh-Linde-Trivedi model [29]

M.R. Setare (✉)
Department of Science, Payame Noor University, Bijar, Iran
e-mail: rezakord@ipm.ir

M.R. Setare
Research Institute for Astronomy and Astrophysics of Maragha, P.O. Box 55134-441, Maragha, Iran

is a typical example, which tries to construct metastable de Sitter vacua in the light of type IIB string theory. Despite the lack of a quantum theory of gravity, we can still make some attempts to probe the nature of dark energy according to some principles of quantum gravity. An interesting attempt in this direction is the so-called "holographic dark energy" proposal [30–52]. Such a paradigm has been constructed in the light of holographic principle of quantum gravity [53, 54], and thus it presents some interesting features of an underlying theory of dark energy. More recently a new dark energy model, dubbed agegraphic dark energy has been proposed [55] (see also [56–58]), which takes into account the Heisenberg uncertainty relation of quantum mechanics together with the gravitational effect in general relativity.

In the present paper, we suggest a correspondence between the new agegraphic dark energy scenario and the phantom dark energy model. We show this phantomic description of the interacting new agegraphic dark energy in non-flat universe, and reconstruct the potential of the phantom scalar field.

2 Interacting Agegraphic Phantom in Non-flat Universe

We consider the non-flat Friedmann-Robertson-Walker universe with line element

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \tag{1}$$

where k denotes the curvature of space $k = 0, 1, -1$ for flat, closed and open universes respectively. A closed universe with a small positive curvature ($\Omega_k \sim 0.01$) is compatible with observations [59, 60]. In this section we obtain the equation of state for the agegraphic energy density when there is an interaction between agegraphic energy density ρ_Λ and a Cold Dark Matter(CDM) with $w_m = 0$. The continuity equations for dark energy and CDM are

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda)\rho_\Lambda = -Q, \tag{2}$$

$$\dot{\rho}_{\rm m} + 3H\rho_{\rm m} = Q. \tag{3}$$

The interaction is given by the quantity $Q = \Gamma\rho_\Lambda$. This is a decaying of the agegraphic energy component into CDM with the decay rate Γ. Taking a ratio of two energy densities as $u = \rho_{\rm m}/\rho_\Lambda$, the above equations lead to

$$\dot{u} = 3Hu\left[w_\Lambda + \frac{1+u}{u}\frac{\Gamma}{3H}\right]. \tag{4}$$

Here, as in Ref. [61], we choose the following relation for decay rate

$$\Gamma = 3b^2(1+u)H, \tag{5}$$

with the coupling constant b^2. Following Ref. [62], if we define

$$w_\Lambda^{\rm eff} = w_\Lambda + \frac{\Gamma}{3H}, \qquad w_m^{\rm eff} = -\frac{1}{u}\frac{\Gamma}{3H}, \tag{6}$$

then, the continuity equations can be written in their standard form

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda^{\rm eff})\rho_\Lambda = 0, \tag{7}$$

$$\dot{\rho}_m + 3H(1 + w_m^{\mathrm{eff}})\rho_m = 0. \tag{8}$$

We use the Friedmann equation to relate the curvature of the universe to the energy density. The first Friedmann equation is given by

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2}[\rho_\Lambda + \rho_{\mathrm{m}}]. \tag{9}$$

We define as usual

$$\Omega_{\mathrm{m}} = \frac{\rho_m}{\rho_{cr}} = \frac{\rho_{\mathrm{m}}}{3M_p^2 H^2}, \qquad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}} = \frac{\rho_\Lambda}{3M_p^2 H^2}, \qquad \Omega_k = \frac{k}{a^2 H^2}. \tag{10}$$

According to the new agegraphic dark energy we have the following relation for energy density [63–66]

$$\rho_\Lambda = 3n^2 M_p^2 \eta^{-2}, \tag{11}$$

where the numerical factor $3n^2$ is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe, and η is the conformal time given by

$$\eta = \int \frac{dt}{a} = \int \frac{da}{a^2 H}. \tag{12}$$

Using (10), we get

$$H\eta = \frac{n}{\sqrt{\Omega_\Lambda}}. \tag{13}$$

Also, using (6), (7), (11), (13), one can obtain the equation of state as

$$w_\Lambda = -\left(1 - \frac{2}{3na}\sqrt{\Omega_\Lambda} + \frac{\Gamma}{3H}\right). \tag{14}$$

Then, using (5), (10) we can rewrite the above equation as follows

$$w_\Lambda = -\left(1 - \frac{2}{3na}\sqrt{\Omega_\Lambda} + \frac{b^2(1+\Omega_k)}{\Omega_\Lambda}\right), \tag{15}$$

where w_Λ can now cross the phantom divide if $\frac{b^2(1+\Omega_k)}{\Omega_\Lambda} > \frac{2}{3na}\sqrt{\Omega_\Lambda}$. This implies that one can generate phantom-like equation of state from an interacting new agegraphic dark energy model in non-flat universe only if $\frac{3nb^2}{2} > \frac{\Omega_\Lambda^{3/2}}{(1+\Omega_k)a}$.

Now we assume that the origin of the dark energy is a phantom scalar field ϕ, so

$$\rho_\Lambda = -\frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{16}$$

$$P_\Lambda = -\frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{17}$$

In this case w_Λ is given by

$$w_\Lambda = \frac{-\frac{1}{2}\dot{\phi}^2 - V(\phi)}{-\frac{1}{2}\dot{\phi}^2 + V(\phi)}. \tag{18}$$

According to the forms of phantom energy density and pressure equations (16) and (17), one can easily derive the scalar potential and kinetic energy term as

$$V(\phi) = \frac{1}{2}(1 - w_\Lambda)\rho_\Lambda, \tag{19}$$

$$\dot{\phi}^2 = -(1 + w_\Lambda)\rho_\Lambda. \tag{20}$$

Differentiating (9) with respect to the cosmic time t, one finds

$$\dot{H} = \frac{\dot{\rho}}{6HM_p^2} + \frac{k}{a^2}, \tag{21}$$

where $\rho = \rho_m + \rho_\Lambda$ is the total energy density. Now, using (2, 3) we get

$$\dot{\rho} = -3H(1 + w)\rho, \tag{22}$$

where

$$w = \frac{w_\Lambda \rho_\Lambda}{\rho} = \frac{\Omega_\Lambda w_\Lambda}{1 + \frac{k}{a^2 H^2}}. \tag{23}$$

Substituting $\dot{\rho}$ into (21), we obtain

$$w = \frac{2/3(\frac{k}{a^2} - \dot{H})}{H^2 + \frac{k}{a^2}} - 1. \tag{24}$$

In a phantom dominated universe $\dot{H} > 0$, from (24) one can easily see that in the $k = 0$, $k = -1$ cases we have $w < -1$, therefore in these cases $w_\Lambda < -1$ as well. For $k = 1$, the necessary condition to obtain $w_\Lambda < -1$ is this: $\dot{H} > \frac{1}{a^2}$.

Using (23, 24), one can rewrite the agegraphic energy equation of state as

$$w_\Lambda = \frac{-1}{3\Omega_\Lambda H^2}\left(2\dot{H} + 3H^2 + \frac{k}{a^2}\right). \tag{25}$$

By substituting the above w_Λ into (19, 20), we obtain

$$V(\phi) = \frac{M_p^2}{2}\left[2\dot{H} + 3H^2(1 + \Omega_\Lambda) + \frac{k}{a^2}\right], \tag{26}$$

$$\dot{\phi}^2 = M_p^2\left[2\dot{H} + 3H^2(1 - \Omega_\Lambda) + \frac{k}{a^2}\right]. \tag{27}$$

In the spatially flat case where $k = 0$ and $\Omega_\Lambda = 1$, (26) and (27) are exactly the same (6) in [67] if we consider $\omega(\phi) = -1$. Similar to the Refs. [67–70], we can define $\dot{\phi}^2$ and $V(\phi)$ in terms of a single function $f(\phi)$ as

$$V(\phi) = \frac{M_p^2}{2}\left[2f'(\phi) + 3f^2(\phi)(1 + \Omega_\Lambda) + \frac{k}{a^2}\right], \tag{28}$$

$$\dot{\phi}^2 = M_p^2\left[2f'(\phi) + 3f^2(\phi)(1 - \Omega_\Lambda) + \frac{k}{a^2}\right]. \tag{29}$$

In the spatially flat case the (28) and (29) are solved only in case of presence of two scalar potentials $V(\phi)$ and $\omega(\phi)$. Here we have claimed that in the presence of curvature term $\frac{k}{a^2}$, (28) and (29) may be solved with potential $V(\phi)$. (To see the general procedure for such type calculations refer to [67–70].) Hence, the following solution is obtained

$$\phi = t, \qquad H = f(t). \tag{30}$$

One can check that the solution (30) satisfies the following scalar field equation

$$-\ddot{\phi} - 3H\dot{\phi} + V'(\phi) = 0, \tag{31}$$

where dots and primes, denote derivative with respect to time and ϕ, respectively. Therefore, by the above condition, $f(\phi)$ in our model must satisfy the following relation

$$3f(\phi) = V'(\phi). \tag{32}$$

On the other hand, using (11), (15), (19), and (20) we have

$$V(\phi) = \left(3 - \frac{\sqrt{\Omega_\Lambda}}{na} + \frac{3b^2(1+\Omega_k)}{2\Omega_\Lambda}\right) M_p^2 H^2 \Omega_\Lambda, \tag{33}$$

$$\dot{\phi} = \left(3b^2(1+\Omega_k) - \frac{2}{na}\Omega_\Lambda^{3/2}\right)^{1/2} M_p H. \tag{34}$$

Using (34), we can rewrite (33) as

$$V(\phi) = 3M_p^2 H^2 \Omega_\Lambda + \frac{\dot{\phi}^2}{2}, \tag{35}$$

or in another form

$$V(\phi) = 3M_p^2 f^2(\phi)\Omega_\Lambda + \frac{1}{2}. \tag{36}$$

Then, from (28, 36), we get

$$\frac{k}{a^2} = 3f^2(\phi)(\Omega_\Lambda - 1) - 2f'(\phi) + \frac{1}{M_p^2}. \tag{37}$$

On the other hand, using (15), (25), and (30), one can obtain

$$\frac{k}{a^2} = 3\Omega_\Lambda f^2(\phi)\left(1 - \frac{2\sqrt{\Omega_\Lambda}}{3na} + \frac{b^2(1+\Omega_k)}{\Omega_\Lambda}\right) - 2f'(\phi) - 3f^2(\phi). \tag{38}$$

Now, using (37), (38) we find

$$\Omega_\Lambda = \left(\frac{na}{2}\right)^{2/3}\left(3b^2(1+\Omega_k) - \frac{1}{M_p^2 f^2(\phi)}\right)^{2/3}, \tag{39}$$

where

$$a = e^{\int f(\phi)d\phi}. \tag{40}$$

Substituting the above Ω_Λ into (36) we obtain the scalar potential as follows

$$V(\phi)=3M_p^2f^2(\phi)\left(\frac{n}{2}\right)^{2/3}e^{\frac{2}{3}\int f(\phi)d\phi}\left(3b^2(1+\Omega_k)-\frac{1}{M_p^2f^2(\phi)}\right)^{2/3}+\frac{1}{2}. \qquad (41)$$

The scalar field potential $V(\phi)$ depends on the function $f(\phi)$ which itself is the derivative of the potential. So this scalar field potential must be consistent with the condition (32).

3 Conclusions

Although a complete theory of quantum gravity has not yet been established, one can make some attempts to investigate the nature of dark energy according to some principles of quantum gravity. The agegraphic and new agegraphic models are such examples. In this paper we have associated the interacting new agegraphic dark energy in non-flat universe with a phantom scalar field. We have shown that the new agegraphic dark energy can be described by the phantom in a certain way. Then a correspondence between the new agegraphic dark energy and phantom has been established, and the potential of the agegraphic phantom has been reconstructed.

Acknowledgement This work has been supported financially by Research Institute for Astronomy and Astrophysics of Maragha, Iran.

References

1. Riess, A.G., et al. (Supernova Search Team Collaboration) Astrophys. J. **607**, 665 (2004)
2. Perlmutter, S., et al. (Supernova Cosmology Project Collaboration) Astrophys. J. **517**, 565 (1999)
3. Spergel, D.N., et al.: Astrophys. J. Suppl. **148**, 175 (2003)
4. Allen, S.W., et al.: Mon. Not. R. Astron. Soc. **353**, 457 (2004)
5. Nojiri, S., Odintsov, S.D.: Phys. Rev. D **68**, 123512 (2003)
6. Nojiri, S., Odintsov, S.D.: Int. J. Geom. Methods Mod. Phys. **4**, 115 (2007)
7. Setare, M.R., Saridakis, E.N.: Phys. Lett. B **670**, 1 (2008). arXiv:0810.3296 [hep-th]]
8. Ratra, B., Peebles, P.J.E.: Phys. Rev. D **37**, 3406 (1988)
9. Wetterich, C.: Nucl. Phys. B **302**, 668 (1988)
10. Liddle, A.R., Scherrer, R.J.: Phys. Rev. D **59**, 023509 (1999). arXiv:astro-ph/9809272
11. Zlatev, I., Wang, L.M., Steinhardt, P.J.: Phys. Rev. Lett. **82**, 896 (1999)
12. Guo, Z.K., Ohta, N., Zhang, Y.Z.: Mod. Phys. Lett. A **22**, 883 (2007)
13. Caldwell, R.R.: Phys. Lett. B **545**, 23 (2002)
14. Caldwell, R.R., Kamionkowski, M., Weinberg, N.N.: Phys. Rev. Lett. **91**, 071301 (2003)
15. Nojiri, S., Odintsov, S.D.: Phys. Lett. B **562**, 147 (2003). arXiv:hep-th/0303117
16. Onemli, V.K., Woodard, R.P.: Phys. Rev. D **70**, 107301 (2004). arXiv:gr-qc/0406098
17. Setare, M.R.: Eur. Phys. J. C **50**, 991 (2007)
18. Feng, B., Wang, X.L., Zhang, X.M.: Phys. Lett. B **607**, 35 (2005)
19. Guo, Z.K., et al.: Phys. Lett. B **608**, 177 (2005)
20. Li, M.-Z., Feng, B., Zhang, X.-M.: J. Cosmol. Astropart. Phys. **0512**, 002 (2005)
21. Feng, B., Li, M., Piao, Y.-S., Zhang, X.: Phys. Lett. B **634**, 101 (2006)
22. Setare, M.R.: Phys. Lett. B **641**, 130 (2006)
23. Zhao, W., Zhang, Y.: Phys. Rev. D **73**, 123509 (2006)
24. Setare, M.R., Sadeghi, J., Amani, A.R.: Phys. Lett. B **660**, 299 (2008)
25. Sadeghi, J., Setare, M.R., Banijamali, A., Milani, F.: Phys. Lett. B **662**, 92 (2008)
26. Setare, M.R., Saridakis, E.N.: Phys. Lett. B **668**, 177 (2008)
27. Setare, M.R., Saridakis, E.N.: arXiv:0807.3807 [hep-th]]
28. Setare, M.R., Saridakis, E.N.: J. Cosmol. Astropart. Phys. **0809**, 026 (2008)
29. Kachru, S., Kallosh, R., Linde, A., Trivedi, S.P.: Phys. Rev. D **68**, 046005 (2003)

30. Cohen, A.G., Kaplan, D.B., Nelson, A.E.: Phys. Rev. Lett. **82**, 4971 (1999)
31. Horava, P., Minic, D.: Phys. Rev. Lett. **85**, 1610 (2000)
32. Thomas, S.D.: Phys. Rev. Lett. **89**, 081301 (2002)
33. Hsu, S.D.H.: Phys. Lett. B **594**, 13 (2004)
34. Li, M.: Phys. Lett. B **603**, 1 (2004)
35. Pavon, D., Zimdahl, W.: Phys. Lett. B **628**, 206 (2005)
36. Enqvist, K., Sloth, M.S.: Phys. Rev. Lett. **93**, 221302 (2004)
37. Ke, K., Li, M.: Phys. Lett. B **606**, 173 (2005)
38. Huang, Q.G., Li, M.: J. Cosmol. Astropart. Phys. **0503**, 001 (2005)
39. Elizalde, E., Nojiri, S., Odintsov, S.D., Wang, P.: Phys. Rev. D **71**, 103504 (2005)
40. Wang, B., Gong, Y., Abdalla, E.: Phys. Lett. B **624**, 141 (2005)
41. Nojiri, S., Odintsov, S.D.: Gen. Rel. Grav. **38**, 1285 (2006)
42. Kim, H., Lee, H.W., Myung, Y.S.: Phys. Lett. B **632**, 605 (2006)
43. Hu, B., Ling, Y.: Phys. Rev. D **73**, 123510 (2006)
44. Li, H., Guo, Z.K., Zhang, Y.Z.: Int. J. Mod. Phys. D **15**, 869 (2006)
45. Setare, M.R.: Phys. Lett. B **642**, 1 (2006)
46. Setare, M.R.: Phys. Lett. B **642**, 421 (2006)
47. Setare, M.R.: Phys. Lett. B **644**, 99 (2007)
48. Setare, M.R., Zhang, J., Zhang, X.: J. Cosmol. Astropart. Phys. **0703**, 007 (2007)
49. Setare, M.R.: Phys. Lett. B **648**, 329 (2007)
50. Setare, M.R.: Phys. Lett. B **654**, 1 (2007)
51. Zhao, W.: Phys. Lett. B **655**, 97 (2007)
52. Li, M., Lin, C., Wang, Y.: J. Cosmol. Astropart. Phys. **0805**, 023 (2008)
53. 't Hooft, G.: arXiv:gr-qc/9310026
54. Susskind, L.: J. Math. Phys. **36**, 6377 (1995)
55. Cai, R.G.: Phys. Lett. B **657**, 228 (2007)
56. Neupane, I.P.: Phys. Lett. B **673**, 111 (2009)
57. Kim, K.Y., Lee, H.W., Myung, Y.S.: Phys. Lett. B **660**, 118 (2008)
58. Zhang, J., Zhang, X., Liu, H.: Eur. Phys. J. **C54**, 303 (2008)
59. Bennett, C.L., et al.: Astrophys. J. Suppl. **148**, 1 (2003)
60. Tegmark, M., et al.: Phys. Rev. D **69**, 103501 (2004)
61. Wang, B., Gong, Y., Abdalla, E.: Phys. Lett. B **624**, 141 (2005)
62. Kim, H., Lee, H.W., Myung, Y.S.: Phys. Lett. B **632**, 605 (2006)
63. Wei, H., Cai, R.G.: Phys. Lett. B **660**, 113 (2008)
64. Kim, K.Y., Lee, H.W., Myung, Y.S., Park, M.I.: Mod. Phys. Lett. A **23**, 3049 (2008)
65. Wu, J.P., Ma, D.Z., Ling, Y.: Phys. Lett. B **663**, 152 (2008)
66. Cui, J., Zhang, L., Zhang, J., Zhang, X.: arXiv:0902.0716 [astro-ph]
67. Nojiri, S., Odintsov, S.D.: Gen. Relativ. Grav. **38**, 1285 (2006)
68. Capozziello, S., Nojiri, S., Odintsov, S.D.: Phys. Lett. B **632**, 597 (2006)
69. Nojiri, S., Odintsov, S.D.: arXiv:hep-th/0611071
70. Nojiri, S., Odintsov, S.D., Stefancic, H.: Phys. Rev. D **74**, 086009 (2006)